Aging population and real exchange rate appreciation: the case of Japan

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Abstract This paper aims at explaining the trending real appreciation experienced by the Yen-US Dollar during the last decade. We develop a two-good overlapping-generation model of a semi-small open economy to highlight the link between the birth rate and the real exchange rate. We find that in a creditor country as Japan, an aging population causes a real exchange rate appreciation due to a positive wealth effect. First, structural parameters are estimated by GMM using quarterly data between 1965-2001. Then, numerical simulations show that the long-run relationship between population growth and real exchange rate is positive and highly nonlinear: the sharp decrease in the rate of population growth may account for about ten percent of the Yen-US Dollar real appreciation.

Key-words: Real exchange rate; overlapping generations; demographics.

Classification J.E.L.: D91; F31; F41.

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1. Introduction

During the last decade, the Japanese economy is characterized by an aging population and a trending real appreciation of the Yen against the US Dollar as depicted by Figure\(^1\). The question addressed in this paper is whether these two features are linked.

Decomposition of the Japan population growth rate between mortality rate and birth rate has been computed from the World Bank WDI 2002 dataset. The summary statistics below exhibits an almost constant mortality rate while the birth rate shows a sharp decreasing over the period which explains major part of the fall in the population growth rate.

**Summary statistics: 1960-2000 (annual)**

<table>
<thead>
<tr>
<th>Series</th>
<th>Mean (annual)</th>
<th>year 1960</th>
<th>year 2000</th>
<th>Label</th>
</tr>
</thead>
<tbody>
<tr>
<td>ε</td>
<td>14.00</td>
<td>17.30</td>
<td>9.43</td>
<td>Birth rate</td>
</tr>
<tr>
<td>p</td>
<td>6.7300</td>
<td>7.60</td>
<td>8.23</td>
<td>Mortality rate</td>
</tr>
<tr>
<td>n</td>
<td>20.7300</td>
<td>24.90</td>
<td>17.66</td>
<td>Pop. growth rate</td>
</tr>
</tbody>
</table>

Such a long appreciation (which starts in 1990) is not frequently observed. The real exchange rate does not exhibit a mean reverting process as the one predicted by PPP. Traditional exchange rate theory would predict a mean reversion of the real exchange rate (RER) and a deterioration of the trade balance when the competitiveness deteriorates, unless the equilibrium RER has been affected by changes in fundamentals. Instead, the Japanese economy runs current account surplus despite the loss in competitiveness. The aim of this paper is to investigate whether the fall in the rate of population growth could explain the departure of the real exchange rate from its PPP level.

\(^1\)Figure 1 uses the following data:

Real Exchange Rate = (Yen/$)\times(\text{US Domestic Price Index} / \text{Japanese Domestic Price index}), base 1=1995 (left scale)

Pop. Growth = Japanese Population Quarterly Growth Rate (right scale).
The theoretical relationship between demographics and the real exchange rate has been very few explored in the recent literature. Gente (2001) shows that in two-sector two-period overlapping generations model, a fall in the birth rate leads to a long-run RER appreciation. In this setting, the young work and save whereas the old, retired, consume the proceed of their savings. As a result an aging population consumes more. Since supply of the domestic good increases less than demand, the exports decrease and the RER appreciates.

Demography is more often used to explain capital flows than real exchange rate behavior. Higgins (1998) shows that the age structure of a population affects significantly the capital flows. Nevertheless, Andersson and Österholm (2001) found empirically a significant relation between the real exchange rate and the demography for Sweden without theoretical framework. In the same way, Andersson and Österholm (2003) supports the same theoretical relations on a panel of 25 OECD countries between 1971 and 2002. Assuming that old agents consume a higher share of services than young agents, Braude
(2000) tests the following assumption: an older population is associated with a more appreciated exchange rate since the consumption of non-tradable goods is higher. Evidence on a panel of 98 countries between 1970 and 1990 supports this assumption.

Our approach consists in two steps. First, we develop an OLG model - with uncertain lifetime expectancy à la Yaari-Blanchard-Weil (1985) - to explore theoretically the link between the birth rate and the RER. With selfish agents, the proportion of new-born in the total population is a determinant of the aggregate wealth. Nevertheless, the change in the financial wealth, following a fall in the birth rate, will depend on the net financial position of the domestic country. In a creditor country like Japan, we obtain that the financial wealth will increase after a fall in the birth rate. As a result, the consumption increases too, causing the real exchange rate appreciation\(^2\).

In a second step, considering the rate of population growth as a proxy of the birth rate\(^3\), we explore empirically this relation. We estimate structural parameters of the theoretical model using GMM on quarterly data over the period 1965Q1-2001Q4. We compute the long run RER and calibrate the model: evidence supports ours theoretical results. We find that the sharp decrease (around one-third between 1965 and 2001) in the rate of population growth in Japan may account for about 10\% of the RER appreciation of the Yen-US Dollar between 1965 and 2001.

Section 2 develops the model. Section 3 investigates the long-run effects of a fall in the birth rate. Section 4 expose econometric implementation and results. Finally, Section 5 provides some conclusions.

\(^2\) We would obtain opposite results in the debtor country case that is a fall in the birth rate would lead to a financial wealth reduction and hence a RER depreciation.

\(^3\) We study a fall in the rate of population growth from a permanent decrease in the birth rate. According to the evidence, the birth rate is a good proxy of the rate of population growth, the death rate being almost constant during the last decades. Notice that an increase in the risk of dying has exactly opposite results.
2. The Model

We describe a two-good overlapping-generation model of a semi-small open economy. We assume that the country is big enough to influence its exports price but sufficiently small in the world economy, as Sen and Turnovsky (1989a, 1989b, 1990), to take the world interest rate $\bar{r}$ as given. The economy consists of cohorts of heterogeneous agents and a representative firm. The firm produces a unique commodity from a neoclassical production technology $F$. The agents allocate their consumption between two substitutable commodities: a domestically produced good $x$ and an imported good $y$. $R$ denotes the relative price of the imported good in terms of the domestic good. All quantities are expressed in units of the domestic good. Under these conventions, a rise in $R$ means a RER depreciation. Under perfect capital mobility, the domestic interest rate is defined according to the Interest Rate Parity (IRP) relation: $r = \bar{r} + \bar{R}/R$. Production is either sold to residents who consume $x$ and invest $I$, or exported $Z(R)$.

2.1. The Individual’s Consumption Behavior

As Buiter (1988), we use the simplest version of the Yaari-Blanchard model of consumer behavior [Yaari (1985), Blanchard (1985)] separating out the birth and death rates. During his lifetime, each consumer faces an age independent instantaneous probability of death $p$ and maximizes hence his expected utility. Consequently, the effective discount rate will be the sum of the rate of time preference $\beta$ and the instantaneous probability of death $p$. Population is growing at the constant rate $n$ equal to the birth rate $\varepsilon$ minus the probability of death $p$. The initial size of the population is normalized at unity $N(0) = 1$. Since we assume no connections between generations’ wealth, an insurance company offers annuities to insure individuals against the risk of dying $p$. Assuming free entry, the insurance premium must equal the instantaneous probability of death $p$. As a result, the return on financial assets is the sum of the world interest rate and the insurance premium $\bar{r} + p$.

We denote by $a(s, t)$ the time-$t$ savings of an agent born at time $s$. Each agent is born
with zero non-human wealth \( a(s, s) = 0 \) and offers labor unelastically. Instantaneous preferences are defined over the two goods according to

\[
c(x, y) = x^\alpha y^{1-\alpha}, \quad 0 < \alpha < 1
\]

(2.1)

The optimization problem of an agent born at time \( s \) as of time \( t \geq s \) can be decomposed into an intra- and an inter-temporal problems. First, the agent allocates his resources to consumption and savings. Formally, the intertemporal program is

\[
\max_{c(s, z)} \int_t^\infty \log c(s, z) e^{-(p + \beta)(z-t)} dz
\]

s.t. \( \frac{da(s, z)}{dz} = (r(z) + p) a(s, z) - \pi(z)c(s, z) + w(z) \quad \forall z \geq t \) \hfill (2.2)

\[
\lim_{z \to \infty} a(s, z) e^{-\int_z^t (r(u)+p) du} \geq 0, \quad a(s, t) \text{ given}
\]

where \( w(z) \) is the age independent wage earned at time \( z^4 \), \( c(z) \) is the composite consumption good defined in (2.1) and \( \pi \) its price index. Secondly, the agent shares its composite consumption \( c \) between the two goods \( x \) and \( y \). We show in Appendix 1 that \( \pi = R^{1-\alpha} \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)} \), and \( x = \alpha \pi c = \alpha (1-\alpha)^{-1} Ry \).

The Euler equation of this problem is

\[
\frac{dc(s, z)}{dz} = \left( r(z) - \frac{\pi'(z)}{\pi(z)} - \beta \right) c(s, z) \hfill (2.3)
\]

with \( \frac{\pi'(z)}{\pi(z)} = (1-\alpha) \hat{R}(z) / R(z) \).

Equation (2.3) is specific to an economy with two goods [Dornbusch (1983)]. It gives the individual consumption path resulting from the maximization of the intertemporal objective (2.2). The spending function \( \pi c \) is obtained by integrating Euler’s equation and using the intertemporal budget constraint together with the NPG condition

\[
\pi(t)c(s, t) = \gamma [a(s, t) + h(t)] \hfill (2.4)
\]

where the propensity to consume out of real wealth is \( \gamma = \beta + p \), and \( h(t) = \int_t^\infty w(\sigma) e^{-\int_\sigma^t (r(u)+p) du} d\sigma \) is the expected human wealth of the consumer at date \( t \).

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4Generalizing to age dependent can be done following Blanchard (1985).
2.2. The Representative Firm

The representative firm produces one good, with a linear homogeneous technology $F$. This good is sold in quantity $X$ to domestic consumers. It is still used for investment $I$ entirely of domestic origin\(^5\). The remaining part is exported according to the ad-hoc exports function $Z(R)$ with $Z' > 0$, i.e. a real depreciation increases the quantity of the domestic good exported. Let $\bar{k} \equiv K/\theta L$ the capital per efficiency labor unit and $f$ be the intensive production function such that $f \left( \bar{k} \right) \equiv F(K/\theta L, 1)$ where $F$ has standard concavity and constant return properties. We assume a Harrod’s neutral technical progress $\theta$ which has a constant growth rate $g$. First order conditions, taking into account the labor market equilibrium ($L(t) = N(t) = e^{nt}$), give

$$r(t) = f' \left( \bar{k}(t) \right) - \delta \tag{2.5}$$

$$\bar{w}(t) = \left[ f \left( \bar{k}(t) \right) - \bar{k}(t) f' \left( \bar{k}(t) \right) \right] \tag{2.6}$$

with $\delta > 0$ the rate of capital depreciation.

The marginal productivity of capital net of depreciation is equal to the domestic interest rate. The marginal productivity of labor is equal to the individual wage. The capital stock is entirely of domestic origin but depends on the RER fluctuations through domestic interest rate $r(t)$.

2.3. Aggregation

Let a capital letter $S$ stand for the aggregate variable $s$: aggregate variables are obtained by summing individual variables weighted by the number of individuals alive in each cohort at time $t$: $S(t) = e^{-pt} \int_{-\infty}^{t} s(\sigma, t) e^{-\varepsilon \sigma} d\sigma$. Thus, $A(t) = e^{-pt} \int_{-\infty}^{t} a(\sigma, t) e^{-\varepsilon \sigma} d\sigma$ and $H(t) = e^{-pt} \int_{-\infty}^{t} h(\sigma, t) e^{-\varepsilon \sigma} d\sigma$ or in the differential form when time subscripts are

\(^5\)We assume that investment uses a domestic good to prevent capital from adjusting instantaneously to its optimal long-run level despite the small economy assumption. In a long-run analysis, this assumption doesn’t matter but could serve in further investigation to consider dynamics.
\[ \dot{\tilde{A}} = rA + wL - \pi C \]  \[ \dot{\tilde{H}} = (r + \varepsilon) H - wL \]  \hspace{1cm} (2.7)

Since population and labor productivity are growing, an aggregate variable \( S \) is expressed per efficiency labor units denoted by \( \tilde{s} \). As the instantaneous probability of dying is constant, the propensity to consume \( \gamma \) is age independent. From (2.4),

\[ \pi(t) \tilde{c}(t) = \gamma \left( \tilde{a}(t) + \tilde{h}(t) \right) \]  \hspace{1cm} (2.8)

Using \( x = \alpha \pi c \), the demand for the domestic good is

\[ \tilde{x} = \alpha \gamma (\tilde{a} + \tilde{h}) \]  \hspace{1cm} (2.9)

and the wealth equations in (2.7) become

\[ \dot{\tilde{a}} = (r - g - n) \tilde{a} + \tilde{w} - \tilde{x}/\alpha \]  \[ \dot{\tilde{h}} = (r - g + p) \tilde{h} - \tilde{w} \]

Substituting the dynamic equations (2.7) in (2.9), we have

\[ \dot{\tilde{x}} = (r - g - \beta) \tilde{x} - \varepsilon \gamma \alpha \tilde{a} \]  \hspace{1cm} (2.10)

Since agents are born with zero financial wealth, changes in domestic good consumption depends on financial wealth. Indeed, the last term of equation (2.10) appears in the special case of disconnected generations. The smaller is the birth rate (\( \varepsilon \)) the less domestic consumption variation depends on the level of financial wealth. If \( \varepsilon \) tends to zero, changes in the domestic consumption would depend only on the gap between \( r \) and \( \beta \) as in the infinitely lived generations setting.

### 2.4. Macroeconomic equilibrium

Under the perfect capital mobility assumption, the relation (2.5) links the optimal capital stock to the domestic interest rate and hence the real exchange rate variation is

\[ \dot{\tilde{R}} = R \left[ f'(\tilde{k}) - (\delta + \bar{r}) \right] \]  \hspace{1cm} (2.11)
The equality between uses and resources in domestic good is a second relation linking the optimal capital stock and the RER. Hence, capital accumulation is given by

\[ \dot{k} = f \left( \tilde{k} \right) - \tilde{x} - \tilde{z} (R) - (\delta + g + n) \tilde{k} \]  

(2.12)

where \( \tilde{z} (R) = Z (R) / \theta L \). Financial wealth \( \tilde{a} \) is the sum of foreign assets expressed in units of the domestic good \( R \tilde{b} \) and the capital stock \( \tilde{k} \). Unless otherwise slated, quantities are expressed in units of the domestic good. The consumption of the domestic good is given by

\[ \dot{x} = (r - g - \beta) \tilde{x} - \varepsilon \gamma \alpha \left[ R \tilde{b} + \tilde{k} \right] \]  

(2.13)

The domestic good consumption depends both on the spread between the domestic interest rate and the rate of time preference and on financial wealth as soon as the birth rate is positive.

2.5. Dynamics

We deduce the current account as \( \dot{b} = \left[ \dot{a} - \dot{k} - R \tilde{b} \right] / R \). The dynamic system consists of domestic equilibrium, IRP condition, domestic consumption, foreign assets and capital stock accumulation. We substitute equation (2.5) for the domestic interest rate and we have

\[ \dot{R} = R \left[ f' \left( \tilde{k} \right) - (\delta + \tilde{r}) \right] \]

\[ \dot{k} = f \left( \tilde{k} \right) - \tilde{x} - \tilde{z} (R) - (\delta + g + n) \tilde{k} \]

\[ \dot{x} = \left[ f' \left( \tilde{k} \right) - (\beta + g + \delta) \right] \tilde{x} - \varepsilon \gamma \alpha \left[ R \tilde{b} + \tilde{k} \right] \]

\[ \dot{b} = (\tilde{r} - g - n) \tilde{b} + \frac{1}{R} \left[ \tilde{z} (R) - \frac{1 - \alpha}{\alpha} \tilde{x} \right] \]

This dynamic system has four variables \( \tilde{b}, \tilde{k}, R, \tilde{x} \), two of these \( R \) and \( \tilde{x} \) being forward ones. This system is not separable because from (2.12) the domestic good is both used for consumption, exports and investment implying that dynamics of capital and foreign assets are linked.
2.6. Steady State

The long-run equilibrium exists when $\bar{r} \in [0, r_p]$ with $r_p = g + (n + \beta)/2 + ((n - \beta)^2 + 4\varepsilon\gamma(1 - \alpha))/2$. Then, this long-run equilibrium is unique and satisfies

$$\bar{r} + \delta = f'(\bar{k}^*)$$

(2.14)

$$\bar{x}^* = \alpha \Gamma \left[ f(\bar{k}^*) - \bar{k}^* f'(\bar{k}^*) \right]$$

(2.15)

$$R^*\bar{b}^* = \left( \frac{\bar{r} - g - \beta}{\varepsilon\gamma} \right) \Gamma \left[ f\left(\bar{k}^*\right) - \bar{k}^* f'(\bar{k}^*) \right] - \bar{k}^*$$

(2.16)

$$\bar{z}(R^*) = (\bar{r} - g - n)\bar{k}^* + (1 - \alpha \Gamma) \left[ f\left(\bar{k}^*\right) - \bar{k}^* f'(\bar{k}^*) \right]$$

(2.17)

with $R^*b^*$ the net foreign assets expressed in units of the domestic good and the long-run propensity to consume out of wealth $\Gamma = \varepsilon\gamma [\varepsilon\gamma - (\bar{r} - g - \beta)(\bar{r} - g - n)]^{-1} > 0$.

A star * denotes the steady-state values. Long-run capital intensity is fully determined by the equalization between the returns on capital: $r(t) = \bar{r}$. Long-run human wealth is the discounted wage. Long-run capital stock is determined by standard marginal productivity conditions, from which we obtain the wage, the value of production and human wealth. Long-run consumption is a function of the wage. The relative price satisfies the equality between domestic supply and the sum of domestic consumption and exports.

Long-run financial wealth $\tilde{a}^*$ depends both on the birth rate $\varepsilon$ and on the gap between the domestic time preference and the world interest rate $\bar{r} - \beta$. The long-run solutions (2.14)-(2.17) lead to the following

$$\tilde{a}^* = \left( \frac{\bar{r} - g - \beta}{\varepsilon\gamma} \right) \Gamma \left[ f\left(\bar{k}^*\right) - \bar{k}^* f'(\bar{k}^*) \right]$$

(2.18)

$$\tilde{k}^* = \left[ f\left(\bar{k}^*\right) - \bar{k}^* f'(\bar{k}^*) \right] \frac{\bar{r} + \varepsilon}{\bar{r}}$$

(2.19)

According to (2.18), financial wealth can be positive or negative\(^7\), depending on the gap between the world interest rate and the domestic rate of time preference. This property

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\(^6\)This condition states that $\Gamma\alpha < 1$ that is the share of domestic good in consumption is sufficiently low to prevent exportations from being negative.

\(^7\)This result is based on the assumption that the wage is age independent [e.g. Matsuyama (1987)].
is a characteristic of the disconnected generations framework [e.g. Blanchard (1985)]. Nevertheless, to study the link between the birth rate and the real exchange rate in the case of Japan, we consider hereafter that the rate of time preference is sufficiently low faced with the world real interest rate and hence the country is a creditor vis-à-vis the rest of the world.

Once the steady-state levels of financial wealth \( \tilde{a}^* \) and capital stock \( \tilde{k}^* \) are given, net foreign assets expressed in units of the domestic good \( R^*\tilde{b}^* \) are obtained as a residual. When the world interest rate tends to the rate of time preference, financial wealth is zero. Consequently, the entire domestic stock of capital is financed by foreign debt

\[
-R^*\tilde{b}^* = \tilde{k}^* \tag{2.20}
\]

More generally, the stock of foreign assets is an increasing function of the spread between the world interest rate and the rate of time preference\(^8\). If the world interest rate were unchanged, a decrease in \( \beta \) would entail an increase in foreign assets when \( \tilde{r} > g + \beta \).

### 2.7. Long run relations between productivity growth, world interest rate and RER

A rise in productivity \( \theta \) increases the supply of domestic good and the agents income. However, since agents allocate a \( 1-\alpha \) share of their consumption spending to the imported good, an increase in productivity leads to a rise in exports meaning that the RER depreciates. Of course, the scale of this positive effect depends on the share of domestic goods in total consumption spending.

The consequences of a world interest rate change are less trivial and depend on the

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\(^{8}\text{The long-run financial wealth is positive (negative) when the world interest rate exceeds (is lower than) the rate of time preference. More precisely, if } \tilde{r} < g + \beta \text{ the country is a net debtor. Whereas we always have } \tilde{r} > g + \beta \text{ when the country is a net creditor vis-à-vis the rest of the world.}\)
level of the domestic time preference. From equation (2.17) we obtain

$$\frac{\partial \tilde{z}(R^*)}{\partial \tilde{r}} = \left( \tilde{k}^* + \tilde{r} \frac{d\tilde{k}^*}{d\tilde{r}} \right) - \alpha \frac{\partial \Gamma}{\partial \tilde{r}} \tilde{w}^* - (1 - \alpha \Gamma) \tilde{k}^* f'' \frac{d\tilde{k}^*}{d\tilde{r}}$$

The first term could be considered as unimportant since it results from the assumption that investment is entirely of domestic origin and we will consider hereafter that this negative effect is largely dominated.

The second term depends on the level of the time preference: $\partial \Gamma / \partial \tilde{r} = (\Gamma^2 / \varepsilon \gamma) \left( 2 (\tilde{r} - g) - \beta - n \right)$. Japan is a creditor country and hence $\tilde{r} - g > \beta + n$: a rise in $\tilde{r}$ leads to an increase in the long run propensity to consume $\Gamma$ resulting from a higher return on financial wealth. This second term is negative meaning that a rise in $\tilde{r}$ tends to generate a real appreciation.

Finally, the third term is negative since we have assumed for steady state existence that $\alpha \Gamma < 1$. To sum up, in the long-run relation, $n$ and $\theta$ should have positive influence on the RER whereas the link between $\tilde{r}$ and $R^*$ is less obvious since it could be decomposed as one positive effect and two negative effects.

### 3. Long-run theoretical effects of a fall in the rate of population growth

This setting with no intergenerational link between agents is useful to investigate the effects of a fall in the rate of population growth $n$. Japan experiences such a fall since the beginning of the 70’s, mainly as a consequence of the birth rates’ drop. The question addressed in this article is to account the influence of this ageing population on the RER. Thus, we investigate here the long-run consequences of a fall in $\varepsilon$ in this semi-small open economy model. We consider first the theoretical aspect to put it further in an econometric framework.

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9 Gente and Leon Ledesma (2003) provide a detailed analysis on the long-run consequences of a world interest rate shock in such an OLG model.

10 We can notice that a fall in $\varepsilon$ and a fall in $p$ have opposite qualitative effects. Indeed, a fall in $\varepsilon$ means that less agents born with zero non-human wealth enter in the economy instead of a rise in $p$ means that more agents with positive financial wealth leave the economy.
The long-run equilibrium is given by equations (2.14)-(2.17). We can show that \( \bar{r} - g < \beta + \varepsilon \) is a sufficient\(^{11}\) condition for the equilibrium to be saddle path stable (see Appendix 2). Thus, this model enables to study both the alternative cases \( \bar{r} > g + \beta \) and \( \bar{r} < \beta + g \). Nevertheless, since we are overall interested in explaining the Yen/USD real exchange rate, we investigate only the case where \( \bar{r} > g + \beta \) to describe the behavior of a creditor country.

At the steady state, all variables per efficiency labor units are constant. A permanent fall in the rate of population growth \( n \) corresponds to a lower rate of long-run economic growth. The long-run level of the capital per efficiency labor units \( \tilde{k}^* \) is not affected by a fall in the rate of population growth. So do the production and the wage. As the introduction mentioned, we analyze a fall in the rate of population growth as a decrease in the birth rate \( \varepsilon \).

A fall in \( \varepsilon \) does not influence what we could call the short-run propensity to consume out of wealth \( \gamma = p + \beta \). Nevertheless, when we look at the steady state reduced form equation (2.15), a fall in \( \varepsilon \) influences the long-run propensity to consume out of wealth \( \Gamma \). This change in \( \Gamma \) means that agents consume a higher (resp. a lower) proportion of their constant wage in a creditor (resp. debtor) country

\[
\frac{\partial \Gamma}{\partial \varepsilon} = -\frac{\Gamma^2 (\bar{r} - g - \beta) (p + \bar{r})}{\varepsilon \gamma}
\]

We can notice that \( \partial \Gamma / \partial \varepsilon \leq 0 \) when \( \bar{r} \geq g + \beta \). The variation of \( \Gamma \) emerges because in this model an ageing population leads to an increase in the financial wealth per capita

\[
\frac{\partial \bar{a}^*}{\partial \varepsilon} = (\bar{r} - g - \beta) \gamma \frac{\partial \varepsilon}{\partial \varepsilon} - \Gamma \frac{\gamma}{(\varepsilon \gamma)^2}
\]

Indeed, in a model with selfish agents, the new-born arrive with a zero non-human wealth. Hence, a fall in \( \varepsilon \) - that is an ageing population - entails an increase of the aggregate wealth

\(^{11}\)This stability condition is usual in Blanchard overlapping generations model of a small open economy. Indeed, if the world were composed by several small economies, world balance requires that \( \beta < \bar{r} - g < \beta + \varepsilon \). Otherwise, the most impatient country would accumulate all the world’s wealth. This result would be counter to the small economy assumption.
per capita $\tilde{a}^*$ and since the capital per capita $\tilde{k}^*$ is constant, the net foreign assets per capita $R^*\tilde{b}^*$ increase ($\partial\Gamma/\partial\tilde{a}^* < 0$ and $\partial\tilde{a}^*/\partial\tilde{a}^* < 0$). Even if the rise in consumption does not seem very realistic for Japan, the model enables to capture the fact that an ageing population saves more and accumulates more foreign assets. This is consistent with the Japanese evidence.

To sum up, when the birth rate decreases, the wealth per capita increases if the rate of time preference is low. Through a positive wealth effect, the consumption increases and then with a constant production, the real exchange rate appreciates and the exports decrease

$$\frac{\partial z(R^*)}{\partial \tilde{a}^*} = -\tilde{k}^* - \alpha \frac{\partial \Gamma}{\partial \tilde{a}^*} \left[ f'\left(\tilde{k}^*\right) - \tilde{k}^* f'\left(\tilde{k}^*\right)\right]$$

(3.1)

The sign of (3.1) is ambiguous because we assumed that investment is entirely of domestic origin. Then, the first term, negative, is due to the fact that a fall in the birth rate reduces the effective depreciation of the capital per capita: maintaining $\tilde{k}^*$ constant requires a lower investment and hence a lower demand of domestic goods. We can notice that under reasonable conditions this first effect seems to be less important than the second one. The second effect of a fall in $\epsilon$ pass through domestic consumption: since the country is creditor vis-à-vis the rest of the world, the aggregate per capita financial wealth increases when less agents - with zero financial wealth - enter the economy. Hence, the demand of domestic good is higher and the RER appreciates. We can notice that if investment were entirely constituted by imported goods, the first term in equation (3.1) would disappear and we would have $\partial z(R^*)/\partial \tilde{a}^* > 0$. We consider further that the second effect is dominant and what we later test the existence of a long-run positive relation between the rate of population growth $n$ and the RER.

Finally, an interesting feature of this model is that the consumption reaction to a fall in the birth rate depends on the spread between the world interest rate and the rate of time preference. Indeed, with such overlapping generations structure à la Buiter (1981), the term $\tilde{r} - g - \beta$ is a key-determinant of the net financial position of the domestic country vis-à-vis the rest of the world. The mechanisms are very intuitive: a fall in $\epsilon$
induces a change in the net *per capita* financial wealth of the domestic country. When the
time preference is low, this change in net financial wealth is positive because less agents
with zero financial wealth enter in the economy. Whereas, when the time preference is
high, this change in net financial wealth is negative because the entering agents - with
zero financial wealth - are richer than the old agents - who have debts -. Then, a fall in 
ε reducing the proportion of new-born in the total population makes the economy poorer 
and hence pushes the consumption down. According to these mechanisms, a fall in the
birth rate ε would lead to a RER depreciation in a debtor country.

4. Econometrics

4.1. Structural parameters estimates

4.1.1. The discrete time structural model

In order to estimate structural parameters and proceed to numerical simulations of the
linearized model, we first approximate the dynamical system in a stochastic discrete time
framework. From equations (2.14) to (2.17), we suggest the following set of equations

\[ E_t R_{t+1} - R_t = R_t \left( a k_t^{a-1} - \delta - r_t \right) \]  (4.1)

\[ E_t x_{t+1} - x_t = x_t \left( a k_t^{a-1} - \delta - \beta - g_t \right) - \alpha \left( \beta + p \right) \left( n_t + p \right) \left( R_t b_t + k_t \right) \]  (4.2)

\[ k_{t+1} - k_t = k_t^a - x_t - c_0 R_t^{c_1} - k_t \left( 1 + \delta + g_t + n_t \right) \]  (4.3)

\[ b_{t+1} - b_t = c_0 R_t^{c_1-1} + b_t \left( r_t - g_t - n_t \right) + \left( \frac{\alpha - 1}{\alpha} \right) \frac{x_t}{R_t} \]  (4.4)

Let \( x_t, b_t, \) and \( k_t \) denote respectively home consumption of domestic goods, net foreign
assets and capital stock, all expressed in labor efficiency units. Extending the theoretical
model, we suppose that world interest rate \( r_t \), population growth \( n_t \) and productivity
growth \( g_t \) are generated by a stochastic process. Moreover, we assume a Cobb-Douglas
technology, so in efficiency units of labor

\[ f(k_t) = k_t^a \quad 0 < a < 1 \]
and the exports function is specified as follows

\[ Z(R_t) = c_0 R_t^{c_1} \quad c_0 > 0, c_1 > 0 \]

Parameters of the model are then \( a, c_0, c_1 \) in the production and exports functions, while \( \beta, \delta, \alpha, p \) denote respectively the rate of time preference, the rate of capital depreciation, the coefficient of the composite consumption good and the probability of death.

### 4.1.2. Data

All data series are extracted from the IMF International Financial Statistics at a quarterly frequency and cover the period from 1960Q1 to 2001Q4. Details of computations and summary statistics of data are given in Appendix 3. In order to construct series of capital stock and technical progress index, we assume a depreciation rate \( (\delta) \) of 10% per year while the parameter of the Cobb-Douglas production function \( (a) \) is fixed at a level of 0.33.

All series (capital stock, external net wealth, domestic consumption of home products, exports) are then divided by the size of population multiplied by the technical progress index.

### 4.1.3. GMM estimates

Estimation of structural parameters using the GMM relies upon the following orthogonality conditions, implied by equations of domestic consumption of home goods and dynamic equation of external net wealth, in which \( V_{it} \) denotes element \( i \) in the vector of instruments used in the estimation:

\[ E_t \left[ \left( \frac{x_{t+1} - x_t}{x_t} - (NMPK_t - \beta - g_t) + \alpha (\beta + p)(n_t + p) \left( R_t \frac{b_t}{x_t} + \frac{k_t}{x_t} \right) \right) \cdot V_{it} \right] = 0 \]

\[ E_t \left[ \left( \frac{b_{t+1} - b_t}{b_t} - \frac{Z_t}{R_t b_t} - (r_t - g_t - n_t) - \left( \frac{\alpha - 1}{\alpha} \right) \frac{x_t}{R_t b_t} \right) \cdot V_{it} \right] = 0 \]

We use growth rate of \( x_t \) and \( b_t \) and ratios of aggregates over \( x_t \) and \( b_t \) in order to remove potential stochastic trends in the series involved.
Since we impose specific values on the depreciation rate ($\delta$) and the parameter $a$ in the Cobb-Douglas function, we define net marginal productivity of capital as $NMPK_t = ak_t^{a-1} - \delta$, with $a = 0.33$ and $\delta = 0.1$ per year. The variable $Z_t$ denotes exports of goods and services, directly used in the regression instead of the exports function.

There are two free parameters to estimate in the above set of equations: $\alpha$, the coefficient in the consumption aggregator, and $\beta$, the rate of time preference, since we set $p = 0.0016825$ at the historical death rate mean (on a quarterly basis) over the 1960-2000 period (see Appendix 3). Due to the cross restrictions implied on the parameter $\alpha$, these two equations are jointly estimated over the period 1965Q1-2001Q4, and the set of instruments used are a constant and two-period lagged values of $r_t, n_t, b_t, NMPK_t, x_t, k_t, g_t$ and $z_t$. Trials have shown that estimates are not much affected by other choices of instruments.

As shown in Table 1, the parameters are very precisely estimated\(^{12}\) and the point estimates are very close to those expected: the estimated value of $\alpha, 0.859$, is close to the 0.8523 historical average value of home goods domestic consumption over total consumption (see Appendix 3) while the estimated value of $\beta, 0.00498$, seems realistic. For instance, estimation of time preference recently carried out by Pagano (2004) in the case of Japan lies in the range (0.0059, 0.00959), depending on the specification used by the author.

Table 1. GMM parameters estimates (1965Q1-2001Q4)

<table>
<thead>
<tr>
<th>Coeff</th>
<th>Estimates</th>
<th>Std.err.</th>
<th>T-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.859</td>
<td>0.0036</td>
<td>237.84</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.00498</td>
<td>0.0012</td>
<td>4.05</td>
</tr>
</tbody>
</table>

In order to achieve estimation of structural parameters, we run the following non-linear ECM type regression for the exports function over the period 1965Q1-2001Q4:

$$Z_t = c_0R_t^{c_1} + c_2 \left(Z_{t-1} - c_0R_{t-1}^{c_1}\right) + e_t$$

\(^{12}\)We use the Newey-West correction for heteroskedastic-consistency and MA(4) disturbances.
where $Z_t$ denotes Japan exports in efficiency units of labor and $e_t$ are residuals. Table 2 summarizes estimation results.

**Table 2. Exports function estimates (1965Q1-2001Q4)**

<table>
<thead>
<tr>
<th>Coeff</th>
<th>Estimates</th>
<th>Std.err.</th>
<th>T-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_0$</td>
<td>0.2985</td>
<td>0.0365</td>
<td>8.18</td>
</tr>
<tr>
<td>$c_1$</td>
<td>0.2339</td>
<td>0.0741</td>
<td>3.15</td>
</tr>
<tr>
<td>$c_2$</td>
<td>0.9589</td>
<td>0.0142</td>
<td>67.63</td>
</tr>
</tbody>
</table>

$Nobs:146$ $R^2: 0.9548$

### 4.2. Long run simulations

Substituting production and exports functions and using estimated values of parameters, numerical long run solutions can be computed from equations 2.14 to 2.17.

We use as parameters values: $a = 0.33, \delta = 0.025, p = 0.00168, \alpha = 0.859, c_0 = 0.2985, c_1 = 0.2339, \beta = 0.00498$, while long-run values for productivity growth rate, population growth rate, and US real interest rates are fixed to their sample means over the period 1960Q1-2001Q4 (see Appendix 3): $g = 0.00458, n = 0.000184, r = 0.00736$.

In a first step, we compute long run RER implied by our theoretical model using different set of values for real interest rate (RIR) and population growth (N) around their sample mean. Results are depicted by Figure 2.

Figure 2 exhibits that long run solution for RER is highly non linear with RIR: no solution exists for RIR values out of the interval $[0.004,0.0085]$ while in this interval, the curve is shaped like an inversely U. Moreover, it is only when the RIR lies between $[0.0065,0.0085]$ - which includes the historical mean value - that we observe the expected positive relationship between population growth and long-run RER: in other cases, the relationship becomes negative.
In a second step, we compute the long-run effect of a fall in population growth on the RER. We first compute the long-run value of the RER using historical means of RIR and productivity growth while the population growth rate is the one observed during the year 1960. Second, we redo computation using the population growth rate observed during the year 2000. The implied long-run RER would be in 2000 at a 91.81% level of his 1960’s value. It results that the sharp reduction in the population rate of growth experienced by the Japanese economy might explain about 10% depreciation of the RER.
5. Conclusion

In this article, we develop a two-good OLG model à la Buiter (1988) of a semi-small open economy in order to explore the link between the rate of population growth and the RER. First, we show that in a creditor country like Japan, a fall in the birth rate leads to a RER appreciation. In a second step, we estimate structural parameters using GMM on quarterly data over the period 1965Q1-2001Q4. From implied value of the long run RER, evidence supports the theoretical results.
APPENDIX

A. Intra and Inter-temporal Trade-Off

We follow Obstfeld and Rogoff (1996). Instantaneous individual preferences are defined over a domestic good $x$ and an imported good $y$ according to:

$$u(x, y) = x^\alpha y^{1-\alpha}$$

where $\pi$ is the consumer price index defined as the minimal spending $q = x + Ry$ necessary for buying a unit of composite good $c$. The variable $R$ is the relative price of the imported good in terms of domestic good. Consequently, $q$ and $\pi c$ are measured in terms of domestic prices.

Formally, the optimization problem of an agent born at $s$ and still alive at date $t > s$ can be decomposed in an intra- and an inter-temporal programs. The inter-temporal program is given by (2.2). The Euler equation indicates the global consumption path:

$$\frac{dc(s, z)}{dz} = \left( r(z) - \frac{\dot{\pi}(z)}{\pi(z)} - \beta \right) c(s, z)$$

(A.1)

Integrating Euler’s equation, the level of consumption in $z$ is:

$$c(s, z) = c(s, t) e^{\int_s^t \rho(\sigma)d\sigma}$$

(A.2)

with $\rho(z) = (r - \dot{\pi}/\pi) - \beta$. Thus, evaluated at market price:

$$\pi(z)c(s, z) = \pi(t)c(s, t) e^{\int_s^t (r(\sigma) - \beta)d\sigma}$$

(A.3)

Hence, microeconomic consumption is a constant share of the real wealth, when utility is logarithmic:

$$c(s, t) = \gamma \frac{a(s, t) + h(s, t)}{\pi(t)}$$

(A.4)

Beyond this intertemporal arbitrage, agents share their consumption spending between the two goods according to:

$$c = \max_{x,y} x^\alpha y^{1-\alpha}$$

s.t.: $Ry + x = \pi c$  

(A.5)
This objective being concave, first order conditions are necessary and sufficient:

\[
\frac{x}{y} = R \frac{\alpha}{1 - \alpha}
\]

(A.6)

Hence, consumer price index is an increasing function of the RER: \( \pi = \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)} R^{1-\alpha} \).

The optimal consumption share between the two goods is:

\[
x = \alpha \pi c \\
R y = (1 - \alpha) \pi c
\]

(A.7)

B. Dynamical system

The dynamical system is:

\[
\begin{align*}
\dot{b} &= (\bar{r} - n) b + \frac{1}{R} \left[ Z(R) - \frac{1-\alpha}{\alpha} \dot{x} \right] \\
\dot{k} &= \theta f \left( \frac{k}{R} \right) - \bar{x} - Z(R) - (\delta + n) \frac{k}{R} \\
\dot{R} &= R \left[ \theta f' \left( \frac{k}{R} \right) - (\bar{r} + \delta) \right] \\
\dot{x} &= \left[ \theta f' \left( \frac{k}{R} \right) - (\beta + \delta) \right] \bar{x} - \varepsilon \gamma \alpha \left[ R \ddot{b} + \ddot{k} \right]
\end{align*}
\]

Around the steady state, the linearized system is:

\[
\begin{pmatrix}
\dot{b} \\
\dot{k} \\
\dot{R} \\
\dot{x}
\end{pmatrix} =
\begin{pmatrix}
\bar{r} - g - n & 0 & \frac{1}{R^2} \left[ \dot{z}'(R^*) + (\bar{r} - n) \dot{b}^* \right] & -\frac{1-\alpha}{\alpha} \frac{1}{R^2} \\
0 & \bar{r} - g - n & -\dot{z}'(R^*) & -1 \\
0 & R^{\ast} \theta f'' \left( \frac{k}{R^*} \right) & 0 & 0 \\
-\varepsilon \gamma \alpha R^* & -\varepsilon \gamma \alpha + \theta f'' \left( \frac{k}{R^*} \right) \bar{x}^* & -\varepsilon \gamma \alpha \dot{b}^* & \bar{r} - \beta
\end{pmatrix}
\begin{pmatrix}
\dot{b} - \dot{b}^* \\
\dot{k} - \dot{k}^* \\
\dot{R} - R^* \\
\dot{x} - \bar{x}^*
\end{pmatrix}
\]

and \textbf{Jac} denotes the Jacobian matrix. Its determinant is:

\[
\text{det Jac} = \theta f'' \left( \frac{k}{R^*} \right) R^{\ast} \dot{z}'(R^*) (\bar{r} + p)(\bar{r} - g - \beta - \varepsilon)
\]

There are two backward variables \( b \) and \( k \). Consequently, saddle path stability requires that two eigenvalues have negative real parts. Therefore \( \text{det Jac} \) must be positive which implies that \( \bar{r} - g < \beta + \varepsilon \).
C. Econometrics appendix

C.1. Data

All data series are extracted from the IMF International Financial Statistics at a quarterly frequency.

*Domestic price index and real exchange rate* US and Japan domestic price levels (DP) are obtained by solving the theoretical relationship linking consumer price index (CPI), import price index (PM) and the non observable domestic price index (DP) with respect to DP: $CPI = PM^{1-\alpha}DP^\alpha$. The parameter ($\alpha$) is computed at each period as one minus the share of imports in total consumption (see below the summary statistics), since the theoretical model rests on the assumption that capital goods are not imported. Real Exchange Rate is then defined as the product of nominal dollar/yen exchange rate and US domestic price index relative to Japan.

**Summary statistics: 1960Q1-2001Q4**

<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$ Japan</td>
<td>0.8523</td>
<td>0.0313</td>
<td>0.765</td>
<td>0.906</td>
<td>0.8407</td>
<td>0.8621</td>
</tr>
<tr>
<td>$\alpha$ USA</td>
<td>0.8927</td>
<td>0.0374</td>
<td>0.815</td>
<td>0.951</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Aggregate capital stock* Japanese capital stock is computed assuming a 10% annual depreciation rate ($\delta = 0.025$ in quarterly frequency) and accumulating the sum of total fixed capital formation and changes in inventory expressed in domestic price units from 1960Q1 to 2001Q4. We assume that initial capital stock was 60% of the GDP, which is not too far from the data produced by Angus Maddison (1995)$^{13}$.

*External wealth* Japan net external wealth is computed accumulating current account balance from 1960Q1 to 2001Q4 and adding revenues of external wealth, the later being defined as the product of nominal 10 years US government bonds, corrected for the nominal exchange rate variation, by the (t-1) stock of wealth. Initial external wealth is

---

$^{13}$Maddison suggests the following capital-output ratios (Machinery and Equipment/GDP) for the year 1950: 0.72 in Japan, 0.64 in the USA and 0.31 in the UK. Due to the measurement problems of capital stock, we retains a more conservative ratio than Maddison’s.
assumed to be zero, as it is found by Lane and Milesi-Ferretti (1999).

*Domestic consumption of home goods* Japan consumption of home goods is defined by the difference between aggregate consumption and aggregate imports.

*World real interest rate* The world real interest rate is computed by substracting a five quarter centered moving average of US domestic annualized rate of inflation to the nominal 10 years US government bonds.

*Technical progress* The Harrod’s neutral technical progress ($\theta_t$) is computed from the Cobb-Douglas specification: let $F(K_t)$ the GDP *per capita* and $K_t$ the capital stock *per capita*, the production function is $F(K_t) = \theta_t K_t^a$. Assuming $a = 0.33$, the productivity index is then: $\theta_t = F(K_t)/K_t^a$. Different trials have shown that the main results are not much affected by alternative values of the parameter $a$.

The following table summarizes sample means of japan productivity and population growth rates and US real interest rate.

**Summary statistics: 1960Q1-2001Q4 and subperiods**

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>g</td>
<td>0.00458</td>
<td>0.00584</td>
<td>0.00354</td>
<td>Productivity growth rate (quarterly)</td>
</tr>
<tr>
<td>r</td>
<td>0.00736</td>
<td>0.00295</td>
<td>0.01125</td>
<td>US Real interest rate (quarterly)</td>
</tr>
<tr>
<td>n</td>
<td>0.00184</td>
<td>0.00271</td>
<td>0.00105</td>
<td>Population growth rate (quarterly)</td>
</tr>
</tbody>
</table>

**References**


